Calculus Name_____

1. Given $x^2y - 2y + 5 = 0$

a. Write an equation of the line tangent when $x = 1$.
b. Find all points where the curve is horizontal, if any.
c. Find all points where the curve is vertical, if any.

a) $y - 5 = 10(x-1)$

b) $\frac{dy}{dx} = \frac{-2xy}{x^2-2} = 0$

$(0, \frac{5}{2})$

c) $\frac{dy}{dx}$ = undefined

$x^2 - 2 = 0$
$+2 \quad +2$
$x^2 = 2$
$x = \pm\sqrt{2}$

Plug into original to find y

$(\sqrt{2})^2 y - 2y + 5 = 0$
$2y - 2y + 5 = 0$
$5 = 0$

$(-\sqrt{2})^2 y - 2y + 5 = 0$
$2y - 2y + 5 = 0$
$5 = 0$

Since there are no y values there are no points with vertical tangents

2. Given $y^3 + 6 = xy$

a. Write an equation of the line tangent when $y = 2$.
b. Find all points where the curve is horizontal, if any.
c. Find all points where the curve is vertical, if any.

a) Point: $(7, 2)$

$2^3 + 6 = 2x$
$8 + 6 = 2x$
$14 = 2x$
$7 = x$

Slope: $\frac{2}{5}$

$y - 2 = \frac{2}{5}(x-7)$

$3y^2\frac{dy}{dx} = x\frac{dy}{dx} + y$

$3y^2\frac{dy}{dx} - x\frac{dy}{dx} = y$

$\frac{dy}{dx} = \frac{y}{3y^2 - x}$

$\frac{dy}{dx} = \frac{2}{3(2)^2 - 7} = \frac{2}{5}$

b) $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{y}{3y^2 - x} = 0$

$y^3 + 6 = xy$

$0^3 + 6 = x(0)$
$6 = 0$
?

There are no points where there is a horizontal tangent.

c) $\frac{dy}{dx}$ = undefined

$\frac{dy}{dx} = \frac{y}{3y^2 - x} = 0$

$3y^2 - x = 0$
$3y^2 = x$

Tells the relationship at the point where there is a vertical tangent.

Original

$y^3 + 6 = xy$
$y^3 + 6 = 3y^2 \cdot y$
$y^3 + 6 = 3y^3$
$6 = 2y^3$
$3 = y^3$
$\sqrt[3]{3} = y$

$3(\sqrt[3]{3})^2 = x$
$3^{\frac{3}{3}} \cdot 3^{\frac{2}{3}} = x$
$3^{\frac{5}{3}} = x$

$(\sqrt[3]{3^5}, \sqrt[3]{3})$